Indian Statistical Institute
Second Semester Back Paper Examination 2004-2005
M.Math II Year
Graph Theory and Combinatorics
Time: 3 hrs   Date:20-07-05   Max. Marks : 100

1. Let E be a 5-(24, 8, 1) design. Show that any two distinct blocks of E have 0, 2 or 4 points in common. Show that if two blocks have four points in common then their symmetric difference is a block.   [30]

2. Let K_9 be the complete graph on 9 vertices. Suppose e of the edges of K_9 are coloured using two colours. If $e \geq 33$ then show that there must be a mono-chromatic triangle (i.e., such that all the edges of the triangle have the same colour.)   [20]

3. Let B be a block of a non-trivial $2 - (v, k, \lambda)$ design. Choose a block $C \neq B$ at random. Find the mean and varience of the random variable $\#(B \cap C)$ ($\#$ denotes size.) Hence prove that
$\lambda(v - 1) \geq k(k - 1)$.   [20]

4. Let G be a graph on n vertices without any clique of size p $(p \geq 2)$. Use induction on n (or some other argument) to show that G has at most $\left(1 - \frac{1}{p-1}\right) \frac{n^2}{2}$ edges.   [30]